Testing predictability criteria in avalanches

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We study a cellular automaton that presents a behavior similar to that of avalanches in sand piles. The size distribution of these events presents a clear separation between small and large avalanches; the former showing a power-law kind of behavior common to these systems. In this article we compare different schemes of possible predictions of the large events. One, using an algorithm proposed by Rosendahl, Vekić, and Rutledge [Phys. Rev. Lett. **73**, 537 (1994)], follows the activities of small avalanches between consecutive large avalanches; others analyze the distribution of time intervals between consecutive large events, and the distribution of small events between consecutive large avalanches. [S1063-651X(96)05710-8]

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I. INTRODUCTION

The predictability of earthquakes is a problem that needs no justification. The knowledge of the details of a fracture of a geological fault, however, is beyond present capabilities either theoretical or experimental. But seismological details aside, it appears clear that an ubiquitous characteristic of earthquakes is that there exist long periods of accumulation of stress, followed by sudden, large, catastrophic events in which a large displacement of the fault occurs, releasing large amounts of accumulated energy. The fault is then ''reset'' and the process repeats itself. A further important characteristic is that such a process of loading and releasing, does not appear to be periodic at all. Compounded with these problems is the lack of enough data that would allow us to have some form of prediction based on an analysis of the statistics of the temporal series of such events; suffice to say that during the last 100 years there have occurred, in the whole world, only about 1000 earthquakes with magnitudes greater than 7 in the Richter scale [1].

In recent years there has been, in the physics community, a resurgence in investigating the earthquake problem [2–5] both with models and with laboratory experiments. Notably, on the experimental side, research has been focused on the study of avalanches in sandpiles with a variety of experimental setups, such as, rotating drums [6–8] or grain-by-grain added sandpiles [9–11]. The obvious relationship with the earthquake dynamics is the fact that the avalanches also show a characteristic (aperiodic) loading and releasing process. An important feature of laboratory or numerical experiments is that, in principle, it is possible to generate arbitrarily large time series and, hence, robust statistics.

In a recent article Rosendahl, Vekić, and Rutledge [9], studying the dynamics of grain-by-grain added sandpiles, proposed an algorithm for prediction of large avalanches based on the statistics of the small avalanches preceding a large one. That is to say, their experiment shows that after adding grains to the pile many small avalanches occur before every large one is generated, with a clear cut separation as to the meaning of ''small'' and ''large'' (see below). Their suggestion is to use the statistics of the occurrence of the small avalanches as predictors of the large ones. However, because of the difficulty of the experiment, Rosendahl, Vekić, and Rutledge report only 11 large avalanches and, therefore, the results of their test cannot be considered conclusive, as Sammis and Carlson [12] have already pointed out. In this article we analyze the results of a cellular automaton that shows the same qualitative features as those of real avalanches [13]; we use it to test different predictability criteria.

Before getting into the details of the prediction methods, we point out that the present cellular automaton behaves, from a ''macroscopic'' point of view, in a remarkably similar way to sandpiles in a rotating cylinder, as shown in Refs. [13,14], and to the classical conical sandpile, as we shall show here. That is, the time behavior of a global variable in the automaton, namely, its mean energy, is quite similar to the time behavior of the angle of the surface in a rotating cylinder [7,13], or the number of grains in the classical conical sandpile [9].

In a previous report [13], we also showed that, if we treat the mean energy of the automaton in a coarse-grained scale so that only large avalanches are relevant, its evolution is accurately described as a stochastic Markov process. There is good evidence, in agreement with actual experiments, that the same is true for the behavior of the mean angle of the surface of sand piles in rotating cylinders [7]. Thus the study of the automaton has been useful to achieve a better understanding of the statistical dynamics of the sandpile in the rotating drum experiment.

As we shall show, the automaton also has the ability to reproduce the macroscopic behavior of avalanches in the conical sandpiles. That is, it shows the occurrence of large, catastrophic events preceded by many small ones. Due to this similarity, and to the fact that we get much better statistics in less time, we use the automaton to evaluate the pre-

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diction algorithm reported by Rosendahl, Vekić, and Rutledge [9] mentioned above, based on the activity of the system preceding a large event. We also discuss two more different possibilities to predict large events by considering the time elapsed since the last large event occurred, and the number of small events before a large one. See also Ref. [12].

In a sense, the general result of this study is on the negative side. That is, as we shall show, it appears that none of the methods is capable of predicting with acceptable confidence. A combination of all of them would probably be more appropriate. However, we do not believe that the problem lies in the different methods, but rather, the ''failure'' may result because the observed variable (the angle of the pile, the number of grains in the pile, or the energy of the automaton), behaves as a stochastic process. And this, being a consequence of having averaged out the complicated dynamics of the many small degrees of freedom beneath the global variable. These conclusions seem to be supported by the fact, mentioned above, that in a coarse-grained scale the large avalanches are very accurately described as a Markov process. But given the richness of the phenomena at hand, we cannot completely rule out a (nonlinear) deterministic description nor a predictive method based on the time statistics of the events. To add to the elucidation of these different aspects is part of the motivation and relevance of the present study.

II. THE AUTOMATION

Although the description of the automaton has been given in other reports [7,13,15], it is included here for completeness. The automaton is defined by an $N\times N$ lattice where at each site (i,j) a scalar quantity h_{ij}, called here ''energy,'' is defined. At every time step, a fixed amount of energy ϵ is added to a randomly chosen site. If the energy of the site is below a given threshold h_c, time is advanced one unit and another site is chosen at random; otherwise, the site breaks, passes its energy in equal parts to its nearest unbroken neighbors, and has its energy reset to zero. The event or avalanche will propagate if the energy of a site, by means of receiving energy from a broken one, reaches or exceeds the threshold h_c. In this case, the site also breaks, and distributes its energy to its nearest unbroken neighbors. The propagation of the event halts when there is no site with energy above h_c; then, time is advanced one unit. While the avalanche is in progress, time is not advanced at all and broken sites remain in that state. It is of interest to point out that in the experiments of Rosendahl, Vekić, and Rutledge [9] ''time'' has the same meaning as here: ''time'' is advanced until the avalanche has finished. Because all energy quantities can be normalized with h_c, the threshold for rupture is always set to 1.0. Thus ϵ, and any h_{ij} are less than one. Results reported here were obtained with $\epsilon=0.25$, and $N=64$.

There are two mechanisms for the system to lose energy. The first one takes place when sites at the border break. Related to these sites, there are two kinds of neighbors; (a) those laying on the lattice, and (b) hypothetical ones outside of the lattice. The rule allows sites at the border to pass part of their energy to such hypothetical sites too. The second mechanism takes place when a site, not located at the border,

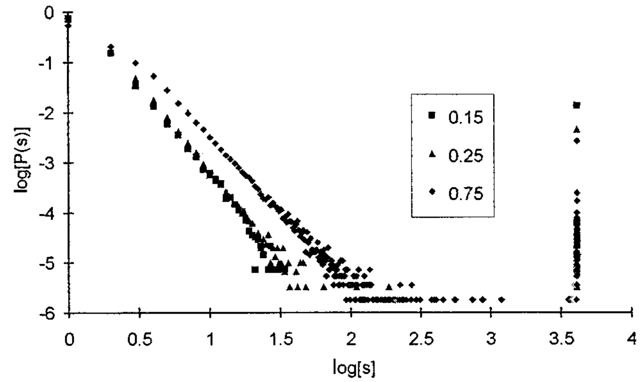

FIG. 1. Normalized size distributions $P(s)$ of avalanches of size s broken sites for $\epsilon=0.15$, 0.25, and 0.75. Base of log is 10.

breaks having no unbroken neighbors able to receive its energy since they have already been broken during the process. In such a case, and due to the fact that no site can remain with its energy above the threshold h_c, the rule forces the site to lose its energy, which is simply dissipated out of the system. While the first energy-dissipation mechanism looks like a natural way for the system to lose energy, resembling the loss of grains in a pile, the second one is inspired by the fracture of a fault in which a stressed site breaks releasing its energy to the surrounding medium [15]. From the point of view of the automaton, these rules are consistent with the condition that broken sites must remain in such a state during the evolution of an avalanche. If this rule is removed, that is, if broken sites can be ''healed'' during the avalanche, then, the automaton displays very different macroscopic behavior from the one we are interested in [15]; as a matter of fact, it then shows self-organized criticality [16], and it has been shown that avalanches in granular media do not belong to this class [17].

It turns out that, with the rules given above, avalanches can be classified as small or large. An avalanche is considered as large if it covers practically the whole system; otherwise it falls into the small category. We find that a large event means that its size is at least 96% of the size of the lattice, and a small event means that it covers at most 25% of the size of the lattice. Events whose sizes fall outside these ranges are seldom seen (see Fig. 1). While the size of the large avalanches is almost independent of the value of ϵ, the size of the small avalanches does depend on it: The smaller the value of ϵ, the smaller the sizes of the small avalanches. For instance, as shown in Fig. 1, for $\epsilon=0.15$ the maximum size of the small events is approximately 35, while with $\epsilon=0.25$ and 0.75 it is 200 and 1200, respectively. We decided to use $\epsilon=0.25$ for the calculations of this work, as a compromise of being small enough to represent a somewhat continuous load of the system, but not too small as far as computing time is concerned, and at the same time having a clear cut separation between small and large avalanches. We also mention that the dissipative mechanism in an isolated nonborder site, occurs essentially during large avalanches only; thus it does not influence the avalanche size distribution.

The ''experimental output'' of the automaton is the time evolution of the total energy, normalized by the size of the system; that is, the mean energy per site given by

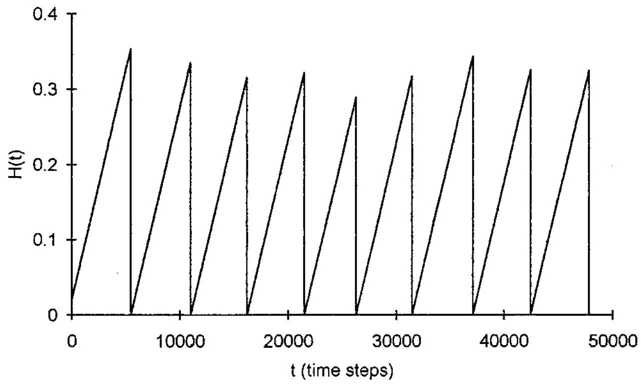

FIG. 2. Mean energy per site $H(t)$ as a function of time step showing buildup periods ending with a large event. Changes in $H(t)$ due to small avalanches cannot be seen at this scale.

$$H(t)=\frac{1}{N^2}\sum_{ij} h_{ij}. \tag{2.1}$$

The evolution of $H(t)$, the macroscopic variable, resembles an irregular sawtooth type of behavior characterized by loading time intervals limited by two large events. During each loading interval, small avalanches occur most of the time; some of them taking away energy from the system and others just redistributing it. Figure 2 shows a typical time sequence of $H(t)$; at the scale of the figure, small events are not visible. As the energy of the system increases, the probability for an avalanche to propagate through the whole system becomes larger. The higher the energy of the system is, the more frequent the small events are, until a large event resets H to zero.

For each avalanche, we determine its size, defined as the number of sites involved in an event, from which we calculate the normalized size distribution. Figure 3 shows such a distribution, for $\epsilon=0.25$, obtained from a run of the automaton of about 7.5×10^8 time steps involving approximately 2.9×10^7 events of any size; 0.5% of them being large events. The separation of small and large avalanches is clear; for this run, there were scarcely 100 avalanches of size in the range of 2% to 97% of the size of the system (that is, between 100 to 4000), which implies that the probability for an event of a size in this range is negligible ($<10^{-7}$). The distribution of small avalanches follows a power-law decay behavior, common to these systems [9,15,18], with an exponent equal to 3.5 ± 0.04

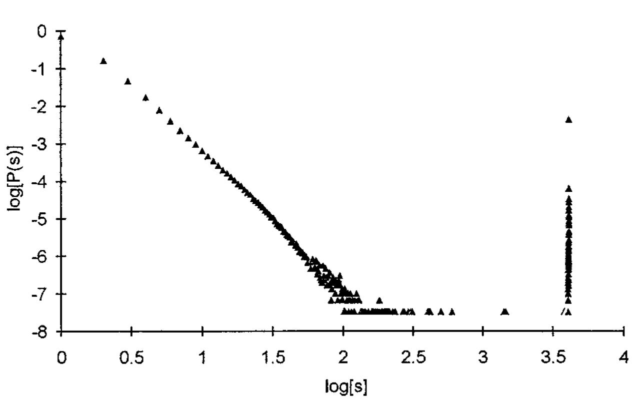

FIG. 3. Normalized size distribution $P(s)$ of avalanches of size s broken sites for $\epsilon=0.25$. Small avalanches scale according to a power-law decay with exponent 3.5 ± 0.04. Base of log is 10.

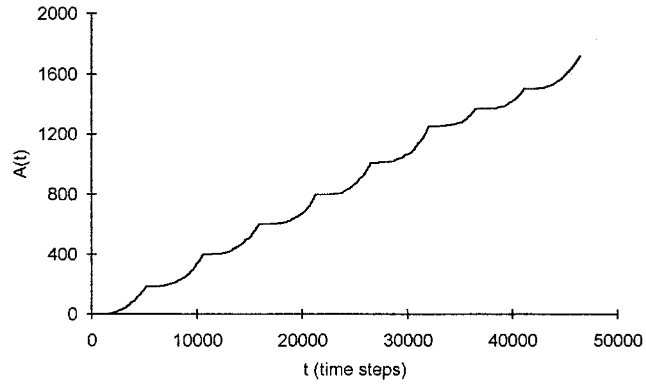

FIG. 4. Total cumulative number of events $A(t)$, regardless of their size, as a function of time step. Discontinuities on the slope indicate the occurrence of a large event.

III. PREDICTION METHODS

Given the irregularity of the occurrence of large avalanches in the experimental situations discussed above and in the automaton as well, the problem seems to be a formidable one as far as the prediction is concerned. But, as Rosendahl, Vekić, and Rutledge observed, the rate of occurrence of small avalanches increases very rapidly as a large avalanche is imminent, and the suggestion is to monitor the small activity in order to see some kind of regularity that would allow us to predict the occurrence of the large events. In this section, we discuss two types of predictive criteria. One is that used by Rosendahl, Vekić, and Rutledge [9] and the other is a probabilistic approach based on the statistics of the process.

A. Activity algorithm

Following Rosendahl, Vekić, and Rutledge [9], we define the total activity $A(t)$ as the cumulative number of events of any size as a function of time, given that at $t=0$ a large event took place. Figure 4 shows a typical output for $A(t)$. There is a remarkable similarity with the corresponding figure shown in Ref. [9]. One can observe that $A(t)$ is a function whose slope changes discontinuously when a large avalanche occurs. That is, after a large event has taken place, the system is left with no energy (in the sandpile, it is left at an angle of repose); thus, it takes some time for all the empty sites to get some energy and small events are not very frequent: the slope of $A(t)$ is very flat after a large event. As time progresses, the system acquires a higher load and the rate of small events also increases, the slope of $A(t)$ becoming steeper, until a large avalanche releases the accumulated energy of the system. And again, the slope of $A(t)$ changes discontinuously.

In order to make an analysis of the activity $A(t)$, and since the loading intervals between consecutive large avalanches have different durations in time, we normalize the latter as follows [9]: Consider a given interval between the

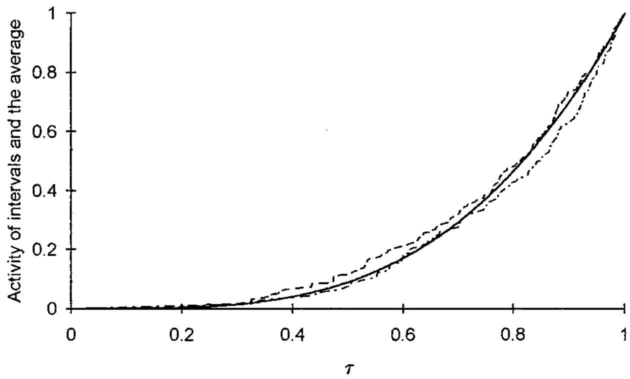

FIG. 5. Activities of two intervals taken from a particular run of the automaton, and the average $a(\tau)$, solid line, of all the interval activities in that run as functions of the normalized time step τ (see text for details). The latter scales according to a power law with slope 3.6 ± 0.04.

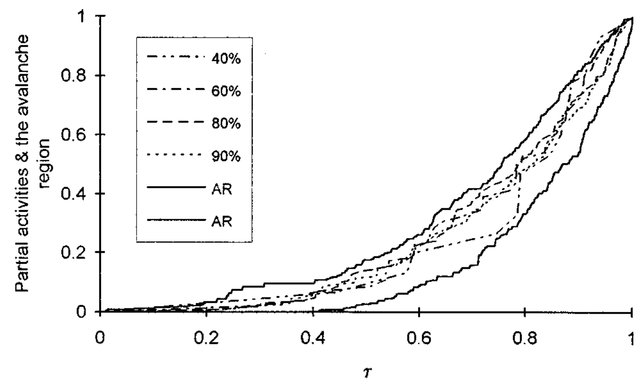

FIG. 7. Partial activities, from one of the interval activities shown in Fig. 5, at 40, 60, 80, and 90% of the interval compared to the avalanche region (AR) (see text for details). The first two are not completely inside the avalanche region.

occurrence of two large avalanches. Let ΔM be the total number of avalanches in such an interval, and let ΔT be its duration. Next, at every time step within the given loading interval, we define $a(\tau)$, as the fraction of events that have occurred up to that time, with τ the time fraction of the interval at that time step. That is, the cumulative number of events is normalized with ΔM, and the time with ΔT. We call $a(\tau)$ the activity of the given interval. Figure 5 shows two particular activities $a(\tau)$ obtained from a run of 5×10^6 time steps involving about 10^3 loading intervals. Also shown in this figure is the average $\overline{a(\tau)}$ of the activities for all the loading intervals. As is the case with the grain-by-grain added sandpile [9], $\overline{a(\tau)}$ also follows a power law; in the present case the exponent is 3.1 ± 0.04.

If we now plot all the activities of the 10^3 loading intervals used to obtain the average $\overline{a(\tau)}$, then, we see that the plane (τ,a) is divided into three regions [9]. One region lies above the upper envelope of all the $a(\tau)$'s, the second region lies below the lower envelope, and the third one—called the avalanche region—is bounded by the two envelopes (see Fig. 6). Since the probability of occurrence for a large event whose size is equal to, or less than, 99.8% ($=4090$) of the system's size is three orders of magnitude less than the probability for an event of the size of the system (Fig. 3), the 10^3 avalanches used here were of size equal to, or greater than, 99.8% of the system's size. The effect that the choice of this threshold has on the avalanche region, and therefore on the results obtained when applying this algorithm, will be discussed in Sec. IV.

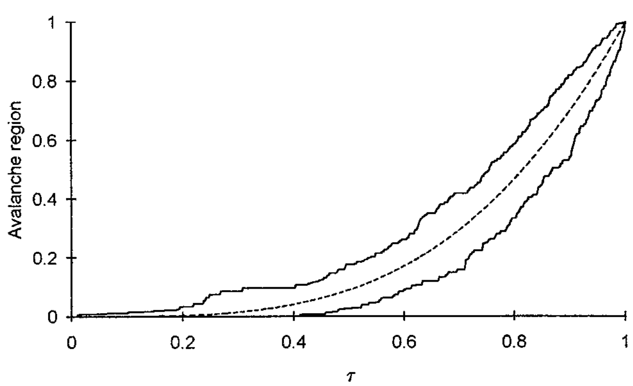

FIG. 6. Envelopes of the interval activities defining the avalanche region for the automaton (see text for details); also shown is the average $\alpha(\tau)$ of the activities, dashed line.

The prediction method consists in following the activity of an interval after a large event has taken place. Then, by using the information given in Fig. 6, decide whether or not a large avalanche will occur in the next time step, thus sounding an alarm. This is done as follows: Let $\Delta M'$ and $\Delta T'$ be the current number of small events and the current time, respectively, after a large avalanche has occurred. Let us now define $\alpha(\tau')$ as the fraction of events that have occurred up to the fraction time τ', with α normalized with $\Delta M'$, and τ' with $\Delta T'$. We call $\alpha(\tau')$ the partial activity of the interval. As the end of an interval comes near, that is, as a large avalanche approaches, $\alpha(\tau')$ should become closer to its corresponding $a(\tau)$. Hence, there may be some threshold to distinguish an interval that is about to end from an interval that is far from its end. In other terms, when plotting $\alpha(\tau')$ one may expect that, if the corresponding interval is near to its end, $\alpha(\tau')$ should be inside the avalanche region.

Following Rosendahl, Vekić, and Rutledge an alarm for a large avalanche is issued if the current $\alpha(\tau')$ is *completely* within the avalanche region. Figure 7 shows partial activities at 40, 60, 80, and 90% of the total duration of the interval corresponding to the first $a(\tau)$ shown in Fig. 5, compared to the avalanche region. It is seen in this figure that for 40% and 60% of the loading interval a small part of the curve is outside the avalanche region just at the very end of the interval; yet, the large avalanche is far from taking place. Now, for the 80% and 90% cases, the curves are completely inside the avalanche region, which just indicates that the large event will take place soon, but it is certainly not imminent.

In order to evaluate how reliable it is to sound the alarm when the partial activity is within the avalanche region, we determined an average prediction for large avalanches in the following way: We let the automaton run for 5×10^6 time steps that involved about 10^3 loading intervals. After every small avalanche in a given loading interval, the correspond-

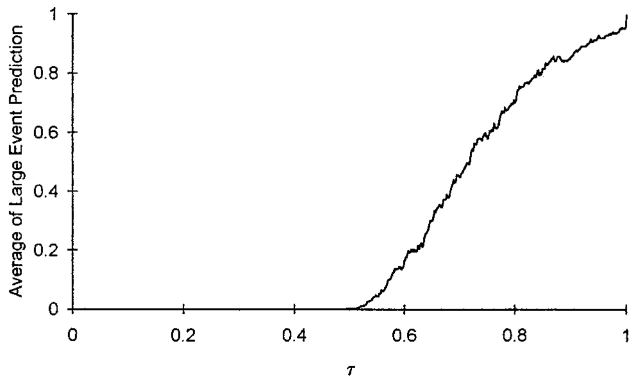

FIG. 8. Average of large event predictions as a function of the normalized time step τ (see text for details). The curve was obtained taking into account 1000 loading intervals.

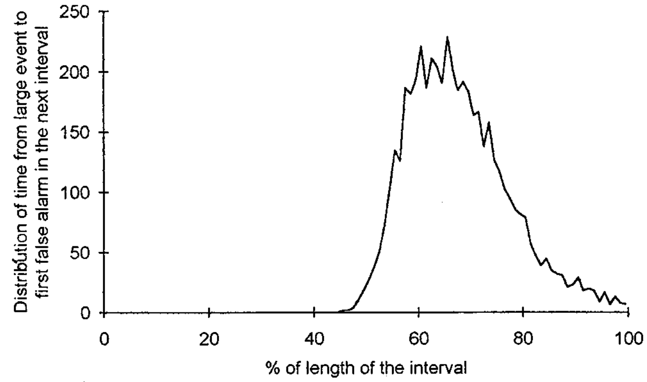

FIG. 9. Distribution of the time elapsed between the last large event and the first false alarm in the next loading interval. Time is given in terms of the percentage of the length of the interval.

ing partial activity $\alpha(\tau')$ is constructed. If $\alpha(\tau')$ is completely within the avalanche region, a YES or 1 value is assigned to that time; otherwise a NO or 0 value is used. At the end of the interval, time is normalized using its time duration. As expected, it turns out that the number of false alarms varies from interval to interval. We consider a false alarm when at time t the partial activity $\alpha(\tau')$ was completely inside the avalanche region, but the large event did not take place at time $t+1$.

With the preceding information, we can calculate the average of all the prediction assignments, as the number of false alarms in a given normalized time (τ), for all the loading intervals, divided by the total number of loading intervals considered. The result is shown in Fig. 8. We can see that in 50% of the loading intervals an alarm was issued when those intervals were at about 75% of their corresponding time duration. Furthermore, the alarm may be issued when the loading interval is as early as approximately 50% of its length. With these results, it seems that the low percentage of large avalanches erroneously anticipated in the experiment carried out by Rosendahl, Vekić, and Rutledge could be due to poor statistics, since they used only 11 large events. That is, the avalanche region may be inaccurate.

A complementary result is to calculate the distribution of times elapsed between the end of a large avalanche and occurrence of the first false alarm in the following loading interval. This distribution is shown in Fig. 9. We see that the first alarm may be sound as early as 50% of the interval, or as late as almost the full interval, with the most probable value of near 70% of the interval.

B. Method based on probability distributions

Since we have at our disposal good statistics for this automaton, we can propose criteria of the prediction of a probabilistic nature. The two distributions that may be used are the distribution of time intervals between consecutive large avalanches $T(t)$ (see Fig. 10), and the distribution of small events during loading intervals, $P(n)$ (see Fig. 11). In the former t is the loading time (i.e., the time between consecutive large avalanches), and in the latter, n is the number of small avalanches in a loading interval. Note that these distributions are constructed independent of each other. The distribution $T(t)$ gives the probability for a large event to occur between times t and $t+1$, given that at time $t=0$ a large event took place, while the distribution $P(n)$ gives the probability that a given interval will have n small events before the large one. From these distributions one can calculate the cumulative probability distributions,

$$C_T(t) = \sum_{i=0}^{t} T(i), \tag{3.1}$$

and

$$C_P(n) = \sum_{n'=1}^{n} P(n'), \tag{3.2}$$

which give the probabilities of occurrence for a large event, between times $t=0$ and t, and n small events, respectively. Then, if the large event has not occurred during the time interval $[0,t-1]$, or after $n-1$ small avalanches, the probability for it to occur at t, or after n, are given by Eqs. (3.1) and (3.2). This gives a different perspective regarding predictions of large events; for instance, it is clear that although in Fig. 7 the partial local activity at 90% is inside the avalanche region, a large avalanche may occur at time $t+1$ with

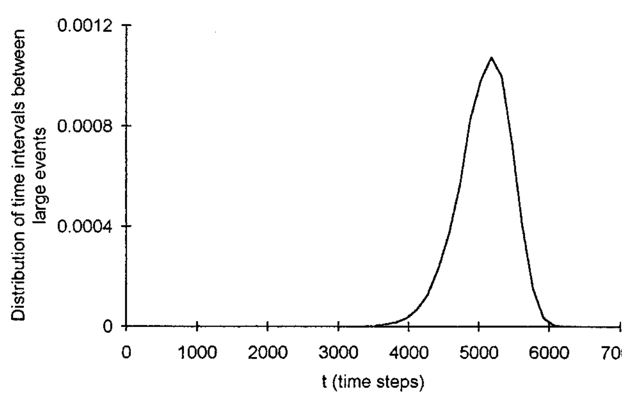

FIG. 10. Normalized distribution of time intervals between consecutive large events.

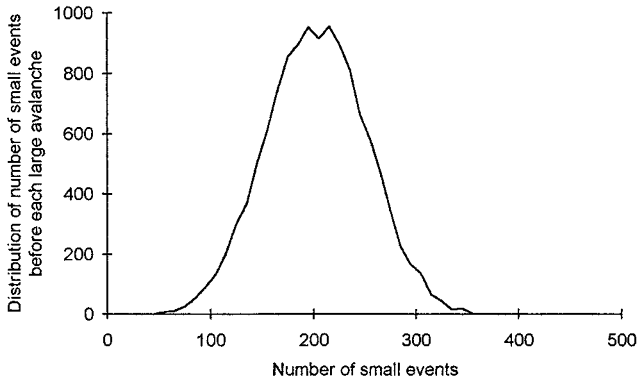

FIG. 11. Distribution of the number of small events that occur before large avalanches.

a probability given by Eq. (3.1). In that particular case, it happened that no large event occurred at time $t+1$, but somewhat later.

Figure 10 indicates the existence of a time window outside of which a large event will never take place. That is, before $t_{\min}\approx 4\times 10^3$ time steps, and after $t_{\max}\approx 6\times 10^3$ time steps, the probability of occurrence of the large event is zero. Once the minimum number of time steps has passed, the large event will occur during a time interval Δt with a probability given by $C_T(t_{\min}+\Delta t)$. Hence, the probability for the large event to occur during the (approximately) next 2×10^3 time steps after $t_{\min}$, equals one. Therefore, predictions for large events using the distribution $T(t)$ should be done as follows: during the first 4×10^3 times steps no alarm is issued since one knows for sure that no large event will occur at any time in that interval; once those first times steps have passed, an alarm for large event will be set at any time as long as the probability of occurrence at the next time step, Eq. (3.1), is also given.

Figure 11 also shows a kind of ''time window'' outside of which a large event never takes place. We see that a minimum number of about 60 small avalanches must occur for the large avalanche to have a nonzero probability of occurrence. Once that number of small events has passed, the large event will be one of the (approximately) next 390 events. Similarly, with $C_P(n)$ we can calculate the probability that the next event will be a large one.

IV. REMARKS

We have studied a cellular automaton whose mean energy $H(t)$, a macroscopic variable, shows a time behavior remarkably similar to those observed in sandpiles. Notably, the energy of the automaton, just as the number of grains in a sandpile, registers the occurrence of large avalanches preceded by a variable large number of much smaller events. Also, the rate of occurrence of the latter, just as in the sandpiles, increases as a large avalanche approaches.

Following the algorithm proposed by Rosendahl, Vekić, and Rutledge [9], we studied the possibility to predict the large events by first defining an avalanche region for this automaton, see Fig. 6, and looking at the time evolution of the partial activity $\alpha(\tau')$ during a loading interval between two consecutive large events. One of the virtues of a numerical model is the capability of generating large sequences of events, leading to much better statistics than in laboratory experiments. For instance, here we built the avalanche region of Fig. 6 from a run that gave rise to approximately 1000 large avalanches. Moreover, we make our predictions not on the same set of data, but during the running time of the automaton [12]. We found that from a total of 1000 new loading intervals, in half of them an alarm for large events was issued when they were at 75% of their time duration. However, the alarm was sometimes turned on during some loading intervals at, as earlier as, approximately 50% of their duration. Furthermore, it may happen that the alarm can be set, or not, several times during the loading interval. Thus the algorithm presents a variable number of false alarms; false in the sense that a large avalanche does not occur at time $t+1$, although the corresponding activity $\alpha(\tau')$ is completely inside the avalanche region at time t.

We indicated in the preceding section that in order to obtain the avalanche region, only events with sizes equal to or greater than 99.8% of the size of the system ($\geqslant$ 4090 sites) were considered as large events, and that about 10^3 of them were taken into account to obtain the avalanche region. We found that if the avalanche region is built with more loading intervals, satisfying the above requirement, it does not change quantitatively. However, if the threshold for an event to be considered as ''large'' decreases, the avalanche region grows, and therefore the number of false alarms in a loading interval increases too. This in itself should not be worrisome; the problem arises because the probability for the occurrence of a large avalanche decreases dramatically with its size. Thus including a very improbable event in the avalanche region can have a strong negative effect. That is, a larger avalanche region causes the first false alarm to be issued, on average, a little earlier and, therefore, the time window determined in Fig. 9 increases. On the other hand, if the avalanche region is built with very few events, it would tend to be smaller than it really is, giving inaccurate results. In Ref. [9], the avalanche region was built with the activity from only 11 loading intervals, and its prediction was verified with the same data. So, the reported low percentage of large avalanches erroneously anticipated may be a consequence of the incompleteness of the avalanche region.

In Ref. [12], Sammis and Carlson reported results for three more algorithms using the same data from Ref. [9]. One of them, a modification of the algorithm by Rosendahl, Vekić, and Rutledge behaves in the same way; the other two algorithms, a time interval method (TI) and a cumulative activity method (CA), report a lower rate of false alarms. The latter two are similar to the ones discussed here (method B in the preceding section). Sammis and Carlson concluded that larger statistics are needed to quantify the relative performance of these algorithms. It is important to point out that in order to avoid that a large event occurs without an alarm, one must choose the minimum threshold from the distributions in Fig. 10 and Fig. 11, respectively. For instance, in the TI method the threshold must be approximately 4000 times steps since no large event occurs before that time. Together with this restriction, it is relevant to know the time window during which the large event is expected to occur. In other words, once the alarm is set, one must know the probability for the large event to take place at a given time t after the alarm, and not only say that a major avalanche is imminent.

From the results presented here it appears that, even with excellent statistics, a probabilistic prediction is the best that one can do at the moment. It also seems obvious that a deterministic description, e.g., a hydrodynamic one, is not possible at the moment. For instance, there is not a generally accepted constitutive relation, such as stress in terms of strains, in granular material. Thus given the small number of macroscopic variables used (the number of grains in the pile or the energy of the automaton), it is not a surprise that the conclusion is that a probabilistic description is the more appropriate. Nevertheless, because of the difficulty of the problem, we find it very important to test different phenomenological ideas or suggestions regarding prediction, such as Rosendahl, Vekić, and Rutledge method, in order to develop the intuition and, with luck perhaps, find some kind of regularity on which to build a solid theory, that at present is lacking.

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